ACHARYA INSTITUTE OF TECHNOLOGY
Bangalore - 560090

# CBCS Scheme

USN | | | | | | | | | | | **15CS36**

## Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
## Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

**Note: Answer any FIVE full questions, choosing one full question from each module.**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

### Module-1

1 a. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound proposition
i) $(p\wedge q)\rightarrow r$ ii) $p\rightarrow(q\wedge r)$ iii) $p\wedge(r\rightarrow q)$ iv) $p\rightarrow(q\rightarrow(\neg r))$ **(04 Marks)**

b. Define tautology. Prove that for any propositions p, q, r the compound proposition $[(p\vee q)\wedge\{(p\rightarrow r)\wedge(q\rightarrow r)\}]\rightarrow r$ is tautology. **(04 Marks)**

c. Establish the validity of the following argument

$$\begin{array}{l}\forall x,[p(x)\vee q(x)]\\ \exists x,\neg p(x)\\ \forall x,[\neg q(x)\vee r(x)]\\ \underline{\forall x,[s(x)\rightarrow\neg r(x)]}\\ \therefore\ \exists x\ \neg s(x)\end{array}$$

**(04 Marks)**

d. Give i) direct proof and ii) proof by contradiction for the following statement. "If 'n' is an odd integer, then n+9 is an even integer". **(04 Marks)**



**OR**

2 a. Define dual of a logical statement. Verify the principle of duality for the following logical equivalence $[\sim(p\wedge q)\rightarrow\sim p\vee(\sim p\vee q)]\Leftrightarrow(\sim p\vee q)$. **(04 Marks)**

b. Prove the following by using laws of logic
i) $p\rightarrow(q\rightarrow r)\Leftrightarrow(p\wedge q)\rightarrow r$
ii) $[\sim p\wedge(\sim q\vee r)]\vee[(q\wedge r)\vee(p\wedge q)]\Leftrightarrow r$. **(04 Marks)**

c. Establish the validity of the following argument using the rules of inference:
$[p\wedge(p\rightarrow q)\wedge(s\vee t)\wedge(r\rightarrow\sim q)]\rightarrow(s\vee t)$ **(04 Marks)**

d. Define i) open sentence ii) quantifiers. For the following statements, the universe comprises all non-zero integers. Determine the truth values of each statement :
i) $\exists x,\exists y\,(xy=1)$ ii) $\exists x,\forall y\,(xy=1)$ iii) $\forall x,\exists y\,(xy=1)$. **(04 Marks)**

### Module-2

3 a. By mathematical induction, prove that
$1^2+3^2+5^2\ldots\ldots.+(2n-1)^2=\dfrac{n(2n+1)(2n-1)}{3}$. **(05 Marks)**

b. For the Fibonacci sequence show that **(05 Marks)**

$$F_n=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

c. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations : i) There is no restriction on the choice ii) Two particular persons will not attend separately iii) Two particular persons will not attend together. **(06 Marks)**

**OR**

4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and /or 7's. **(04 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1}+1$ for $n \geq 2$. **(04 Marks)**

c. i) How many arrangements are there for all letters in the word SOCIOLOGICAL?
ii) In how many of these arrangements A and G are adjacent? In how many of these arrangements all the vowels are adjacent? **(04 Marks)**

d. Find the coefficient of i) $x^9y^3$ in the expansion of $(2x - 3y)^{12}$ ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(04 Marks)**

## Module-3

5 a. Let a function $f : R \rightarrow R$ be defined by $f(x) = x^2 + 1$. Find the images of $A_1=\{2, 3\}$, $A_2 = \{-2, 0, 3\}$, $A_3 = (0, 1)$ and $A_4 = [-6, 3]$. **(04 Marks)**

b. ABC is an equilateral triangle whose sides are of length one cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm. **(04 Marks)**

c. Let f, g, h be functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$

and $h(x) = \begin{cases} 0 & \text{if x is even} \\ 1 & \text{if x is added} \end{cases}$.

Determine $(fo(goh))(x)$ and $((fog)oh)(x)$ and verify that $fo(goh) = (fog)oh$. **(04 Marks)**

d. For A = {a, b, c, d, e} the Hasse diagram for the Poset (A, R) is as shown in Fig Q5(d). Determine the relation matrix for R and Construct the digraph for R **(04 Marks)**

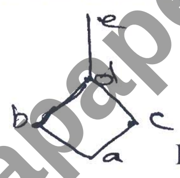

Fig Q5(d)

**OR**

6 a. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine the
i) Number of binary relations on A.
ii) Number of relations from A to B that contain (1, 2) and (1, 5)
iii) Number of relations from A, B that contain exactly five ordered pairs
iv) Number of binary relations on A that contains at least seven ordered pairs. **(04 Marks)**

b. Let A = B = R be the set of the real numbers, the functions $f : A \rightarrow B$ and $g: B \rightarrow A$ be defined by $f(x) = 2x^3 - 1$, $\forall x \in A$; $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}$ $\forall y \in B$. Show that each of f and g is the inverse of the other. **(04 Marks)**

c. Define a relation R on A×A by $(x_1, y_1)\ R\ (x_2, y_2)$ iff $x_1+y_1 = x_2+y_2$, where A ={1, 2, 3, 4, 5}.
i) Verify that R is an equivalence relation on A×A.
ii) Determine the equivalence classes [(1, 3)] and [(2, 4)]. **(04 Marks)**

d. Consider the Hasse diagram of a POSET (A, R) given in Fig Q6(d). If B = {c, d, e} find all upper bounds, lower bounds, the least upper bound and the greatest lower bound of B. **(04 Marks)**

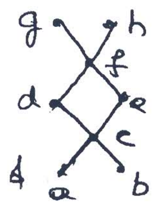

Fig Q6(d)

## Module-4

7 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5. **(04 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(04 Marks)**

c. A girl student has Sarees of 5 different colors, blue, green red, white and yellow. On Monday she does not wear green, on Tuesdays blue or red, on Wednesday blue or green, on Thursday red or yellow; on Friday red. In how many ways can she dress without repeating a color during a week (from Monday to Friday)? **(04 Marks)**

d. The number of affected files in a system 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

**OR**

8 a. In how many ways can one arrange the letters in the word CORRESPONDENTS so that
   i) There is no pair of consecutive identical letters?
   ii) There are exactly two pairs of consecutive identical letters? **(06 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed to four boys $B_1$, $B_2$, $B_3$, and B4. The boys $B_1$ and $B_2$ do not wish to have apple, the boy, $B_3$ does not want banana or mango and $B_4$ refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(05 Marks)**

c. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$ given that $a_1 = 5$ and $a_2 = 3$. **(05 Marks)**



## Module-5

9 a. Define :
   i) Bipartite graph
   ii) Complete bipartite graph
   iii) Regular graph
   iv) Connected graph with an example. **(04 Marks)**

b. Define isomorphism. Verify the two graphs are isomorphic **(04 Marks)**

i)  ii) 

Fig Q9(b)

c. Show that a tree with n vertices has n-1 edges. **(04 Marks)**

d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(04 Marks)**

**OR**

10 a. Determine the order $|V|$ of the graph G = (V, E) in
   i) G is a cubic graph with 9 edges
   ii) G is regular with 15 edges
   iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. **(04 Marks)**

b. Prove that in a graph
   i) The sum of the degrees of all the vertices is an even number and is equal to twice the number of edges in the graph.
   ii) The number of vertices of odd degrees is even. **(04 Marks)**

c. Discuss the solution of Konigsberg bridge problem. **(04 Marks)**

d. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of the optimal tree. **(04 Marks)**